Fourth Semester B.E. Degree Examination, June / July 2014
Advanced Mathematics – II

Time: 3 hrs.   Max. Marks: 100

Note: *Answer any FIVE full questions.*

1 a. Define direction cosine and direction ratio of a line. Hence show that $l^2 + m^2 + n^2 = 1$.
      (06 Marks)

 b. For any cube show that angle between any two diagonals is $\cos^{-1}\left(\dfrac{1}{3}\right)$.
      (07 Marks)

 c. Define plane. Derive equation of plane in general form.   **(07 Marks)**

2 a. Find equation of plane passing through A(-1, 1, 1), B(1, -1, 1) and perpendicular to plane $x + 2y + 2z - 5 = 0$
      (06 Marks)

 b. Show that the line $\dfrac{x-4}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{10}$ is parallel to plane $2x + 2y - z = 6$. Find distance between them.
      (07 Marks)

 c. Show that lines $\dfrac{x-5}{4} = \dfrac{y-7}{4} = \dfrac{z+3}{-5}$ and $\dfrac{x-8}{7} = \dfrac{y-4}{1} = \dfrac{z-5}{3}$ are coplanar. Find point of intersection.
      (07 Marks)

3 a. Find sine and cosine of angle between the vectors $4i + 3j + k$, $2i - j + 2k$.   **(06 Marks)**

 b. Show that points (4, 5, -1), (0, -1, -1), (3, 9, 4), (-4, 4, 4) are coplanar using vector method.
      (07 Marks)

 c. Prove that $\left[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}\right] = 2\left[\vec{a},\vec{b},\vec{c}\right]$.   **(07 Marks)**

4 a. A particle moves along the curve $x = t^3+1$, $y = t^2$, $z = 2t + 5$. Find components of its velocity and acceleration at $t = 1$ in the direction $i + j + 3k$ **(06 Marks)**

 b. Find directional derivative of $x^2 + y^2 + 4xyz$ at (1, -2, 2) in the direction $2i - 2j + k$.
      (07 Marks)

 c. Show that $\operatorname{grad}\left(\dfrac{1}{r}\right) = -\dfrac{\vec{r}}{r^2}$.   **(07 Marks)**

5 a. For any scalar function ϕ show that $\operatorname{curl}(\operatorname{grad}\phi) = 0$.   **(06 Marks)**

 b. If $\vec{F} = \operatorname{grad}\phi$, $\phi = x^2+y^2+z^2+xyz$, find $\nabla \cdot \left(\vec{F}\right)$ and $\nabla \times \left(\vec{F}\right)$ at (1, 1, 1).   **(07 Marks)**

 c. Find a, b, c so that $\vec{F} = (x + y + az)i + (x + cy + 2z)j + (x + 2y - z)k$ is irrotational. Find scalar function.
      (07 Marks)

6 a. Find Laplace Transform if t^n and hence find $L\left(t^{\frac{1}{2}}\right)$. **(06 Marks)**

   b. Find $L\left[e^{2t}\cos 3t + e^{-t}\sin 2t + t\sin t\right]$. **(07 Marks)**

   c. Find $L\left[\dfrac{e^{t}(\cos 3t - \cos t)}{t}\right]$. **(07 Marks)**

7 a. Find $L[\sin t \, \sin 2t \, \sin 3t]$. **(06 Marks)**

   b. Find $L[f(t)]$ where $f(t) = \begin{cases} 1 & 0 < t \le 1 \\ t & 1 < t \le 2 \\ t^2 & t > 2 \end{cases}$. **(07 Marks)**

   c. Find $L^{-1}\left\{\log\sqrt{\dfrac{s+a}{s-b}}\right\}$. **(07 Marks)**

8 a. Find $L^{-1}\left\{\dfrac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}\right\}$. **(10 Marks)**

   b. Solve by Laplace transformation, $\dfrac{d^2 y}{dt^2} + 7\dfrac{dy}{dt} + 10y = 4e^{-3t}$, given $y(0) = 0, y'(0) = -1$. **(10 Marks)**

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